Operations with Rational and Irrational Numbers

9/12 Class Notes

- The set of real numbers consist of *all* numbers on the number line. If it were possible to place a point on the number line for each real number, the line would be completely filled in and extend forever in the negative and in the positive directions.
- As we discussed yesterday, the set of real numbers is made up of two major subsets (smaller groups), called rational and irrational numbers.
- **Rational numbers** - The ratio (fraction) of two integers written in the form of a fraction as long as the denominator is not 0. Consists of integers {... -2, -1, 0, 1, 2...}, fractions (2/3), terminating (.25) and repeating (.$\overline{7}$) decimals, and the roots of "perfect" numbers. ($\sqrt{16}, \sqrt{25}, \sqrt{36}$...) perfect squares
- **Irrational numbers** - Real numbers that cannot be expressed as the ratio of two integers; consists of non-repeating and non-terminating decimals, the roots of numbers which do not calculate to a rational number and pi (π).

Sums and Products of Rational and Irrational Numbers

- Addition of two rational numbers will result in a sum that is a rational number

Examples:

$-3 + 8 =$ 5

$2.9 + 14.8 =$ 17.7

$\frac{1}{5} + \frac{3}{7} =$ $\frac{22}{35}$

$-4 + \sqrt{16} =$ 0

$2.\overline{4} + 1.2 =$ $3.6\overline{4}$

$-\frac{1}{8} + 12 =$ 11.875

- Multiplication of two rational numbers will result in a product that is a rational number.

Examples:

$-3 \times 8 =$ -24

$2.9 \times 14.8 =$ 42.92

$\frac{1}{5} \times \frac{3}{7} =$ .085714... or $\frac{3}{35}$

$-\frac{1}{8} \times 12 =$ -1.5

$2.\overline{4} \times 1.2 =$ $2.9\overline{3}$

$-4 \times \sqrt{16} =$ -16

- Addition of a rational and an irrational number will result in an irrational number.

Examples:

$2 + \sqrt{5} =$ 4.23606...

$3 + \pi =$ 6.14159...

$\sqrt{17} + 0 =$ 4.1231...

$\sqrt{3} + \sqrt{9} =$ 4.73205...

- Multiplication of a non-zero rational and an irrational number will result in an irrational number

Examples:

$2(\sqrt{5}) =$ 4.4721...

$-4 \cdot 2\sqrt{3} =$ -13.8564...

$100 \cdot \pi =$ 314.159...

- The sum or product of two irrational numbers may be rational or irrational.

Examples:

$\sqrt{15} + (-\sqrt{15}) =$ -15

$\sqrt{6} + \sqrt{10} =$ 5.6117...

$\sqrt{7} \cdot \sqrt{7} =$ 7

$\sqrt{5}(\sqrt{3}) =$ 3.8729...